Jet and Rocket Propulsion

AE4451

LECTURE 26

Sedina Tsikata – March 10, 2023

Overview

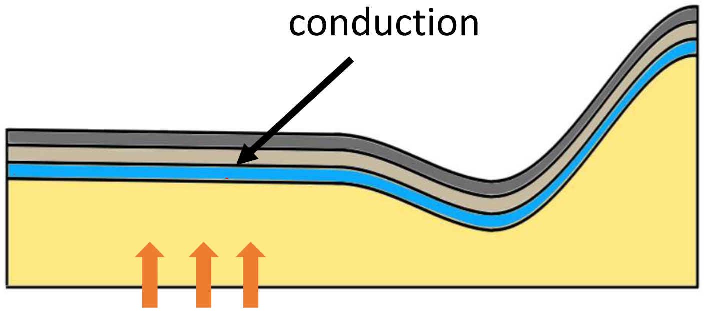
- what we saw in Lecture 24
  - heat transfer
    - types of combustion chamber cooling: regenerative, ablative, radiation, film
    - heat transfer modes: conductive, convective, radiative

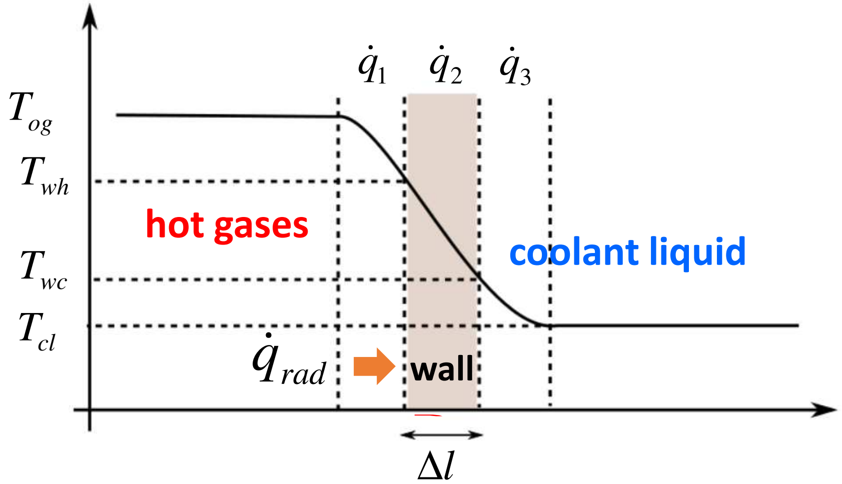
- Lecture 25: midterm exam

- today
  - heat transfer calculations and correlations

Heat transfer

Heat transfer in the combustion chamber and nozzle





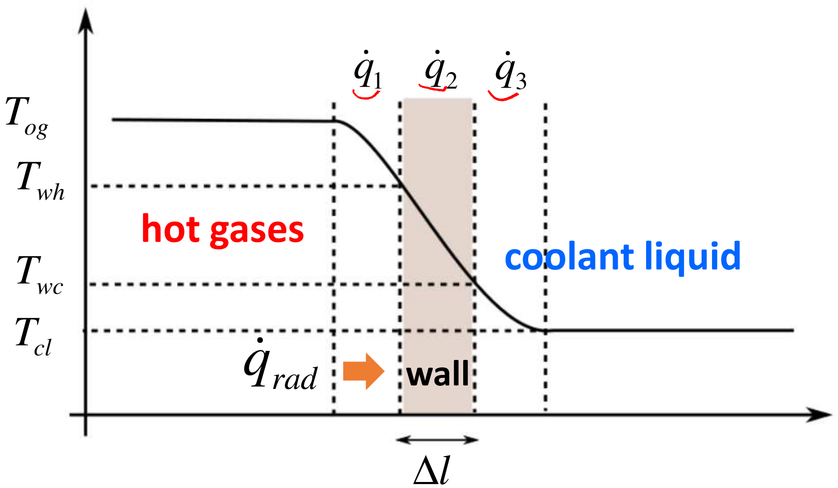
$$\dot{q}_{conv} = \frac{T_{og} - T_{cl}}{\frac{1}{h_g} + \frac{\Delta l}{k} + \frac{1}{h_l}}$$

convective heat flux

- we can also account for radiative heat flux $\dot{q}_{rad}$

Heat transfer

Heat transfer in the combustion chamber and nozzle



accounting for radiative heat flux

$$\dot{q} = \dot{q}_{conv} + \dot{q}_{rad}$$

$$\dot{q} = h_g (T_{aw} - T_{wh}) + \dot{q}_{rad}$$

from before, adiabatic wall temperature: $T_{aw} = rT_{og} + T_{\infty}(1-r)$

for r = 1, $T_{aw} = T_{og}$ adiabatic wall temperature matching hot gas stagnation temperature

$$T_{wh} = T_{og} - \left(\frac{\dot{q}_{conv} - \dot{q}_{rad}}{h_g} \right)$$

substituting expression for $\dot{q}_{conv} = \dfrac{T_{og} - T_{cl}}{\dfrac{1}{h_g} + \dfrac{\Delta l}{k} + \dfrac{1}{h_l}}$

$$T_{wh} = T_{og} - \left(\frac{T_{og} - T_{cl}}{1 + \dfrac{h_g \Delta l}{k} + \dfrac{h_g}{h_l}} \right) + \frac{\dot{q}_{rad}}{h_g}$$

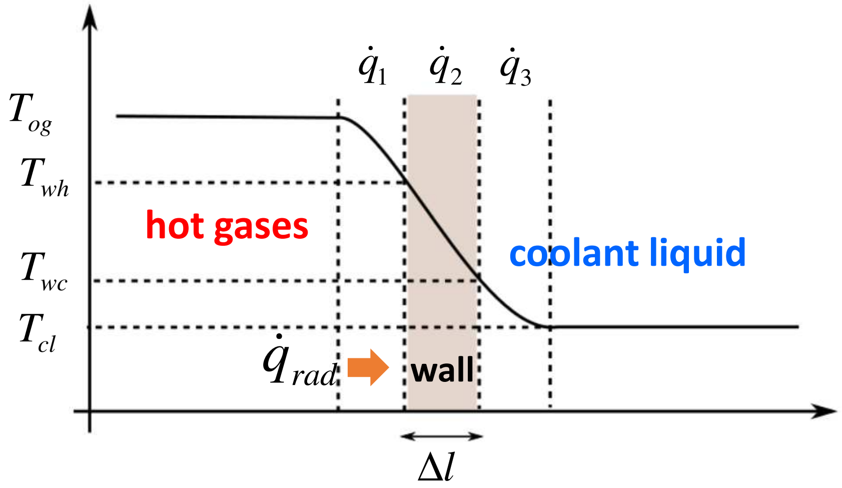
i.e. real wall temperature

Heat transfer

Heat transfer in the combustion chamber and nozzle

$$T_{wh} = T_{og} - \left(\frac{T_{og} - T_{cl}}{1 + \frac{h_g \Delta l}{k} + \frac{h_g}{h_l}} \right) + \frac{\dot{q}_{rad}}{h_g}$$

maximize to get cooler wall



often $\dot{q}_{conv} >> \dot{q}_{rad}$ $\Rightarrow T_{wh} = T_{og} - \left(\frac{T_{og} - T_{cl}}{1 + \frac{h_g \Delta l}{k} + \frac{h_g}{h_l}} \right)$

Heat transfer

Reducing hot wall temperature

$$\frac{T_{og} - T_{cl}}{1 + \frac{h_g \Delta l}{k} + \frac{h_g}{h_l}}$$

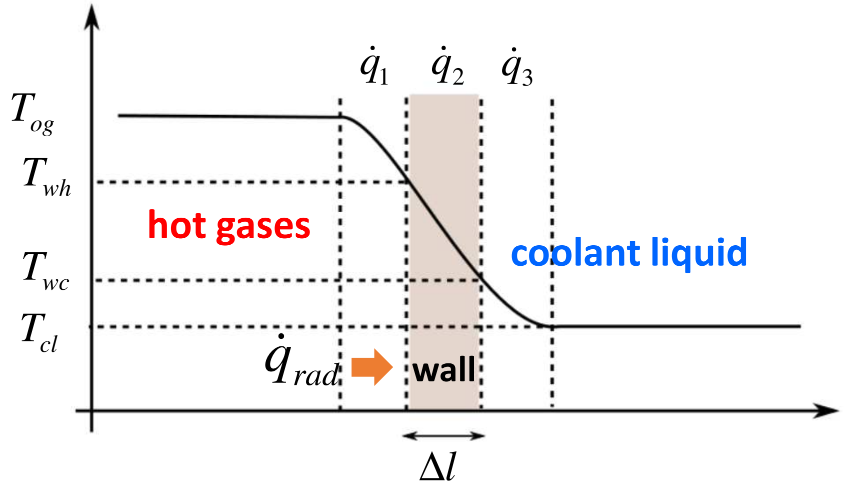
maximize

- lower coolant temperature T_{cl}
  - in combustion chamber, $T_{og} >> T_{cl}$
  - e.g., 3000 K vs 100 – 250 K, so relatively small effect

- use high conductivity wall material (increase k)

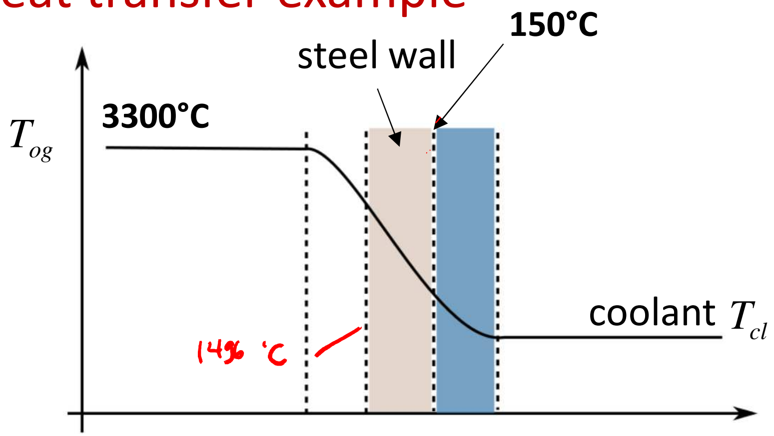
- use thin walls (reduce Δl)
  - structural limits

- lower ratio h_g/h_L
  - reduce heat transfer rate to hot wall, increase rate from cold wall



Heat transfer

Heat transfer example



Consider a given rocket thrust chamber

- outside wall temperature (nozzle throat) = 150°C
- chamber temperature = 3300°C

$$\dot{q}_{throat} = 1.4 \times 10^{7}\ \mathrm{W/m^2}$$

- throat wall material stainless steel, thickness 0.25 cm
- wall conductivity k = 26 W/m.°C

(i) find hot wall temperature at the throat

- assumption: areas equivalent for coolant and hot gases

across metal wall

$$\dot{q}_{s} = \frac{-k}{\Delta l}(T_{wc} - T_{wh})$$

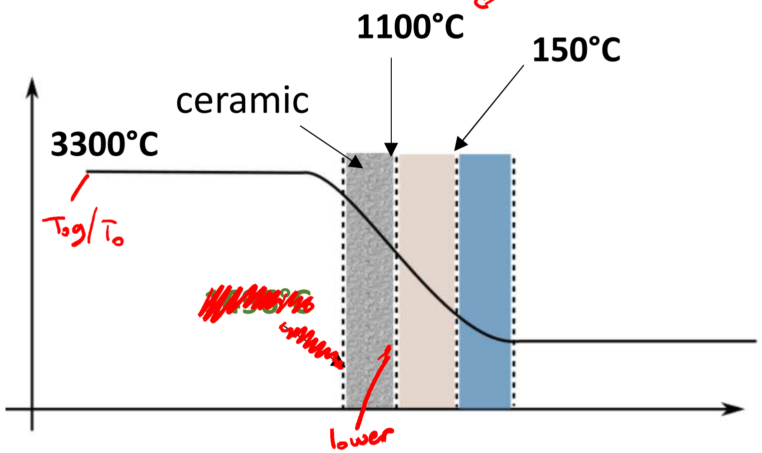
recall, matching fluxes

$$\begin{cases} \dot{q} = h_g (T_{og} - T_{wh}) \\ \dot{q} = \dfrac{-k}{\Delta l}(T_{wc} - T_{wh}) \\ \dot{q} = h_l (T_{wc} - T_{cl}) \end{cases}$$

$$\Rightarrow T_{wh} = T_{wc} + \frac{\dot{q}\Delta l}{k} = 150 + \left(\frac{1.4 \times 10^{7} \times 0.25 \times 10^{-2}}{26} \right)$$

$$= 1496\ ^{\circ}\mathrm{C}$$

Heat transfer example



New scenario:

- to lower temperature, wall is lined with ceramic ceramic conductivity k = 8.65 W/m.°C
- target hot metal wall temperature = 1100°C

(ii) find thickness of ceramic needed

- required heat transfer rate through wall: $\dot{q} = \frac{-k}{\Delta l}(T_{wc} - T_{wh}) = \frac{-26}{0.25 \times 10^{-2}}(150 - 1100)$ (target)

$$= 9.88 \times 10^6 \text{ W/m}^2$$

- hot gas flux $\dot{q} = h_g(T_{og} - T_{wh})$

(need this to determine gas heat transfer coefficient)

$$h_g = \frac{\dot{q}}{T_{og} - T_{wh}} = \frac{1.4 \times 10^7}{3300 - 1496} = 7760.53 \text{ W/(m}^2\text{.°C)}$$

(previous part $\dot{q}$; calculated)

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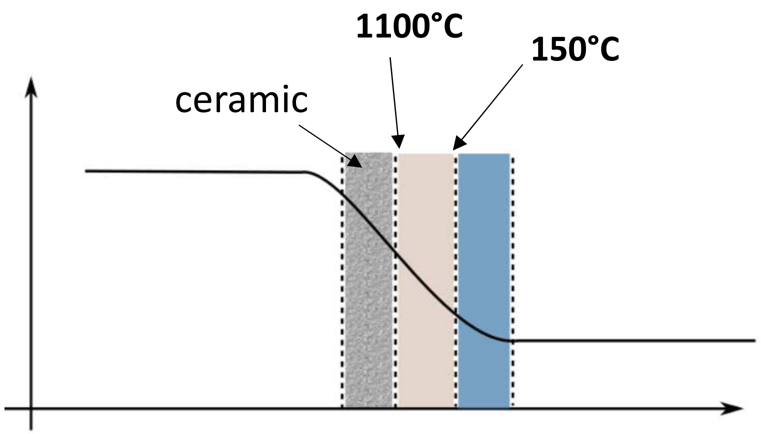
Heat transfer example



(ii) find thickness of ceramic needed

- now find temperature on hot side of ceramic

$$\dot{q} = h_g (T_{og} - T_{wh}) \quad \Rightarrow T_{wh} = T_{og} - \frac{\dot{q}}{h_g}$$

$$T_{wh} = 3300 - \frac{9.88\times10^6}{7760.53}$$

required flux

$$T_{wh} = 2026.89\ °\text{C}$$

- determine ceramic thickness using

$$\dot{q} = \frac{-k}{\Delta l}(T_{wc} - T_{wh})$$

$$\Rightarrow \Delta l = \frac{-k}{\dot{q}}(T_{wc} - T_{wh})$$

$$= \frac{-8.65}{9.88\times10^6}(1100 - 2026.89) \quad = 0.08\ \text{cm}$$

Heat transfer

Convective heat transfer coefficients

- heat transfer problems require information on the convective heat transfer coefficients

  previous example: inferred the heat transfer coefficient using calculated flux

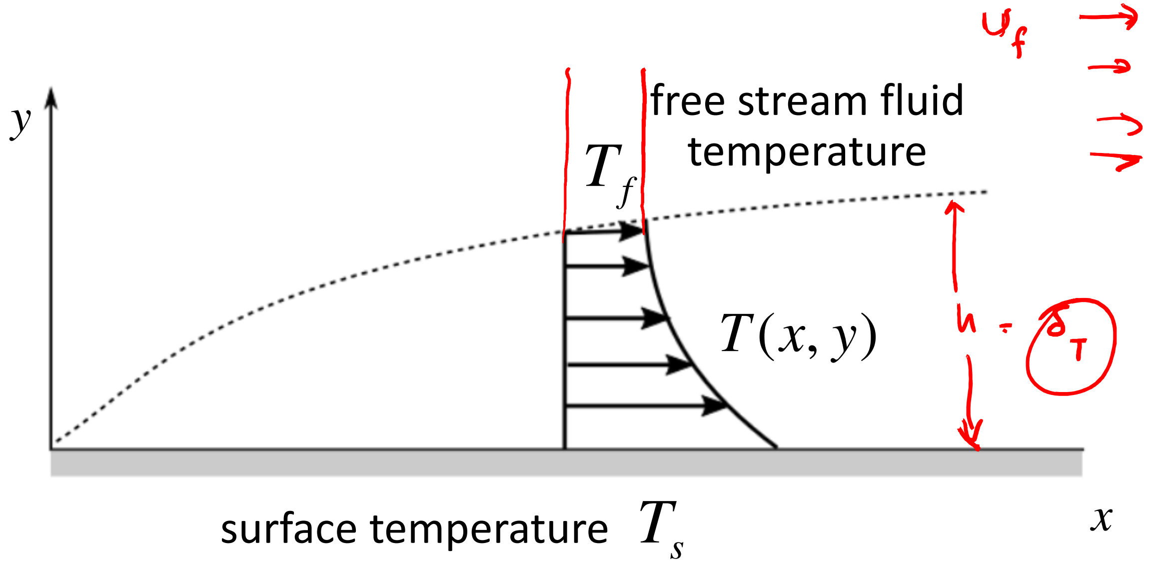
- convective heat transfer

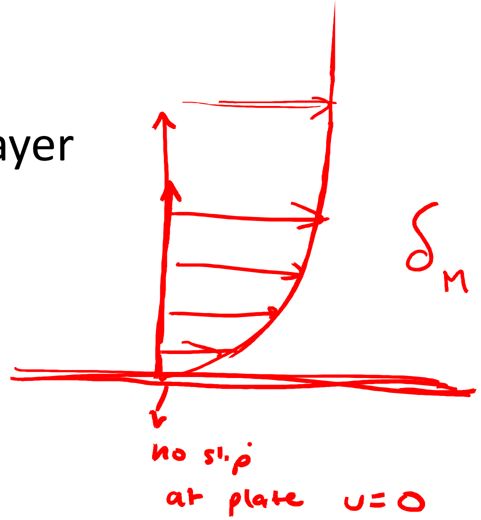
  - due to fluid moving over surface

  - thermal boundary layer develops, analogous to momentum boundary layer

δ_T

thermal boundary layer thickness = where T reaches 99% of T_f





Heat transfer

Convective heat transfer coefficients

- the heat transfer coefficient is a function of *Re* and *Pr*

$$h = f(\mathrm{Re}, \mathrm{Pr})$$ Reynolds, Prandtl numbers

$$\mathrm{Re}_L = \frac{\rho u L}{\mu} = \frac{\rho L}{\nu} \qquad \mathrm{Pr} = \frac{\mu c_p}{k} = \frac{\nu}{\alpha}$$

μ = dynamic viscosity
ν = molecular diffusivity
α = thermal diffusivity

- significance of Pr

= thickness of momentum boundary layer to thermal boundary layer

= ability of fluids to transport momentum + heat via molecular collisions in boundary layers

Pr ~ 1

case for many gases;
momentum layer thickness
~ thermal layer

Pr < 1

case for high conductivity
fluids, e.g. liquid metals

Pr > 1

low conductivity
fluids, e.g. water

Heat transfer

Convective heat transfer coefficients

- for high speed compressible flows: gas kinetic energy important

$$\mathrm{Ec} = \frac{u^2}{c_p (T_w - T_\infty)}$$ Eckert number

u = flow speed
T_w = wall temperature
T_∞ = free stream temperature

- significance of *Ec*

  = ratio of kinetic to thermal energy of flow

  - relevant for hypersonics

- Nusselt number: ratio of convective to conductive heat transfer rates $\mathrm{Nu} \equiv \frac{hL}{k}$
  - convective flows dominant in most cases $\mathrm{Nu} >> 1$
  - if *Nu* known, will know *h*

    $\Rightarrow$ many heat transfer correlations describe *Nu* in terms of *Re* and *Pr*

Heat transfer

Convective heat transfer coefficients

- Stanton number: same purpose as *Nu*

$$\mathrm{St} = \frac{h}{\rho u c_p} = \frac{\mathrm{Nu}}{\mathrm{Re} \times \mathrm{Pr}}$$

can be considered a dimensionless *h* value

Correlations for LREs

- semi-empirical correlations are available for LRE-type geometries and conditions

e.g. hot gas side
Bartz correlation (1965)

$$\mathrm{Nu}_{\mathrm{D}} \propto \mathrm{Re}^{0.8}\mathrm{Pr}^{0.4}$$

D = diameter of thrust chamber assembly at given axial location

e.g. coolant side

$$\mathrm{Nu}_{\mathrm{D_H}} \propto \mathrm{Re}^{0.8}\mathrm{Pr}^{\mathrm{n}}$$

correlations used for fully-developed turbulent flow in a channel

n = 0.33-0.37

D_H = hydraulic channel diameter, generally 4A/P (area / perimeter)

D_H =2HW/(H + W) for rectangular channel

flat plate, laminar (subsonic) flow

$$\mathrm{Nu} = 0.332\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}$$

Heat transfer

Correlations for LREs

- most common in rocket cooling calculations: Seider-Tate correlation

$$\mathrm{Nu} = a\,\mathrm{Re}^{m}\,\mathrm{Pr}^{n}\left(\frac{\mu}{\mu_w}\right)^{b}$$

constants vary between liquids

μ_w = viscosity at wall

Fluid	a	m	n
Methane	0.0023	0.8	0.4
Propane	0.005	0.95	0.4
Kerosene	0.023	0.8	0.4
Liquid hydrogen	0.062	0.8	0.3

- modified form used in rocket cooling jacket calculations: Dittus-Boelter correlation

$$\mathrm{Nu} = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.4}\left(\frac{T_{wc}}{T_{cl}}\right)^{-0.3}$$

reflects high temperature differences between hot wall and fluid

- above forms do not account for wall curvature (nozzle shape); to do so, use Ambrok's equation:

$$\mathrm{St} = \frac{h_g}{\rho_\infty u_\infty c_{p\infty}} = 0.0287\,\mathrm{Pr}^{-0.4}\frac{R^{0.25}(T_\infty - T_w)\mu^{0.2}}{\left[\int_0^x R^{1.25}(T_\infty - T_w)\rho_\infty u_\infty\,dx\right]^{0.2}}$$

where $\mathrm{Pr} = \dfrac{\mu_\infty c_{p\infty}}{k_\infty}$

Heat transfer

Convective heat transfer coefficients

- coolant side: typically fully-developed flow

$$h_{cl}(x) = C\frac{k_{cl}}{D}\left(\overbrace{\frac{\rho_{cl}u_{cl}D}{\mu_{cl}}}^{Re}\right)^{0.8}\left(\overbrace{\frac{\mu_{cl}c_{p,cl}}{k_{cl}}}^{Pr}\right)^{n} \qquad \begin{cases} C = 0.018,\ n = 0.37 \\ C = 0.023,\ n = 0.33 \end{cases}$$

- evaluate fluid properties (k, μ, c_p) at bulk and average liquid temperature at axial location x

for high h_L $\Rightarrow$ high Reynolds number, high k_L and c_{pL}

Heat transfer

Convective heat transfer coefficients

- hot side: not typically fully-developed flow: area changing, flow acceleration
  - changes boundary layer thickness
  - from Bartz, Adv. Heat Trans. (1965): Bartz equation

$$h_g(z) = 0.026 k_g^{0.6} \left(\frac{c_{p_g}}{\mu_g}\right)^{0.4} \frac{\dot{m}^{0.8}}{D^{1.8}} \left(\frac{D_t}{R_c}\right)^{0.1} \sigma$$

R_c = throat radius of curvature

$$h_g \propto \dot{m}_g / D^{1.8}$$

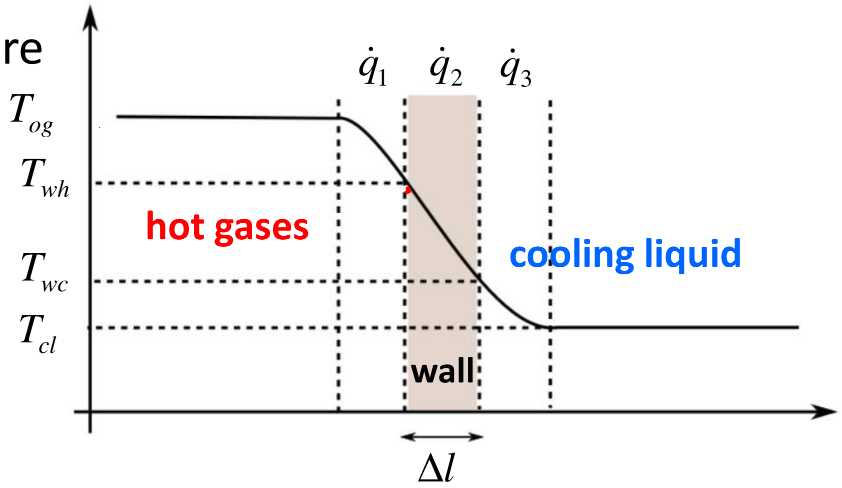
high Q at throat

- evaluate fluid properties at free-stream stagnation temperature

$$\sigma = \left[\frac{1}{2}\frac{T_{wh}}{T_{og}}\left(1+\frac{\gamma-1}{2}M^2\right)+\frac{1}{2}\right]^{0.2n-0.8}\left(1+\frac{\gamma-1}{2}M^2\right)^{-0.2n}$$

n ~ 0.8 for diatomics

n = temperature dependence of μ

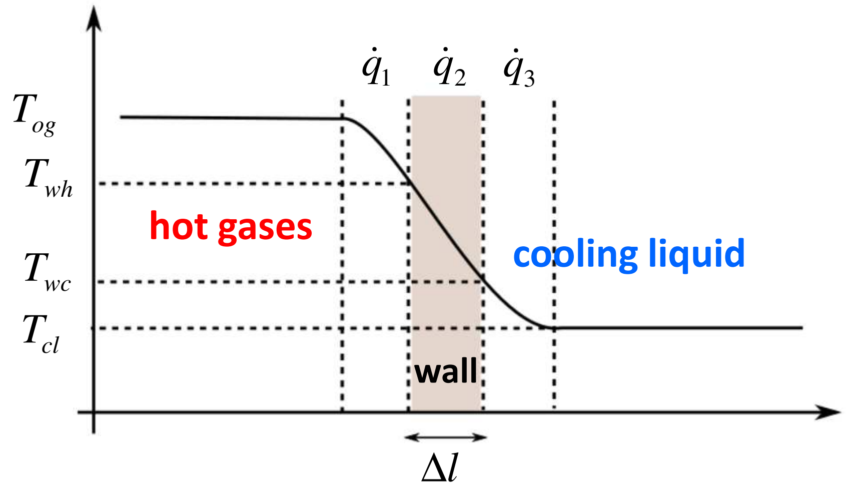


Heat transfer

Convective heat transfer coefficients: compressible effects, origin of σ

- for high M (> 0.7 – 0.8)
  - increase in static temperature of gas in boundary layer as gas slows
  - heat conduction through gas driven by temperature gradient

$$h_g(z) = 0.026 k_g^{0.6} \left(\frac{c_{p_g}}{\mu_g} \right)^{0.4} \frac{\dot{m}^{0.8}}{D^{1.8}} \left(\frac{D_t}{R_c} \right)^{0.1} \sigma$$



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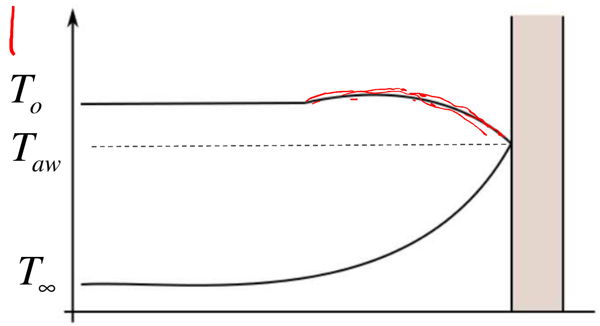
Convective heat transfer coefficients: compressible effects

- we saw before that the adiabatic wall temperature differs from the free stream stagnation temperature



- heat transfer back into free stream from hot, slow moving fluid near wall
- heat transfer in reality not driven by T_o, but by a "recovery temperature" T_r

$$T_r < T_o$$ smaller than To due to viscous dissipation

- r depends on type of boundary layer $r = f(\Pr)$

Flow type	Recovery factor *r* variation
Laminar couette	$\Pr$
Laminar free boundary layer	$\sqrt{\Pr}$
Turbulent free boundary layer	$\Pr^{1/3}$

$$T_r = T_\infty\left(1 + \frac{\gamma - 1}{2} r M_\infty^{\ 2}\right)$$

Heat transfer

Axial heat transfer solution

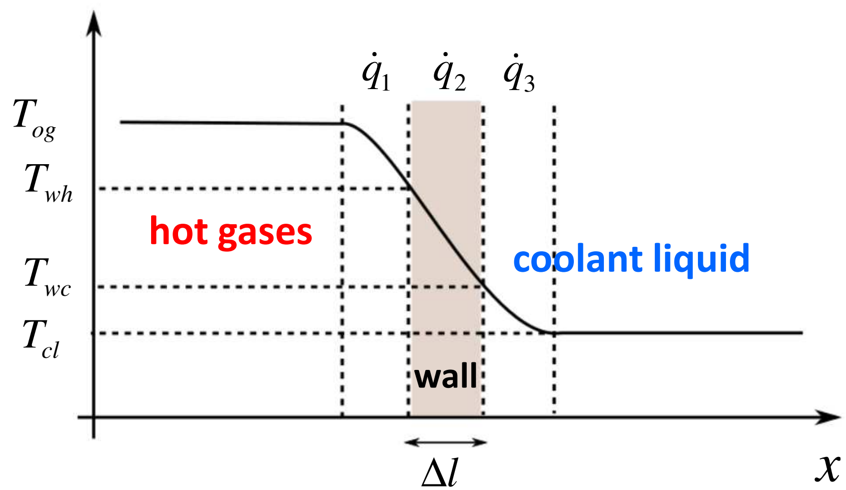
- Goal: determine how thermal conditions vary along *x*, axis of thrust chamber assembly

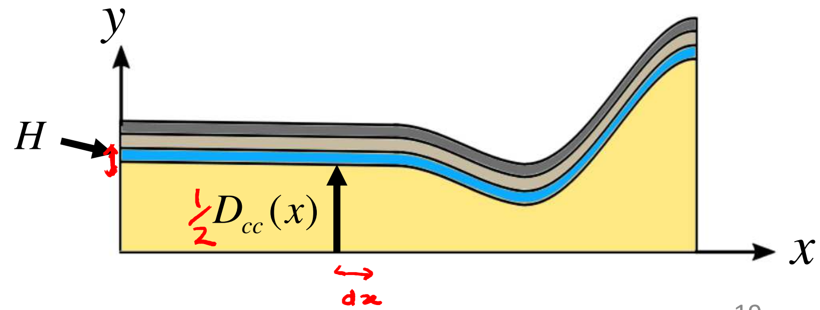
approach - combine previous results with:

- gas energy equation (1D) $\dot{m}_g c_{p,g} dT_{og}(x) = \dot{q}(x)\pi D_{cc}(x)dx$
- coolant energy equation (1D) $\dot{m}_{cl} c_{p,cl} dT_{cl}(x) = \dot{q}(x)Hdx$
- boundary conditions $T_{cl}(x_{\max});\quad T_{og}(x_{\min})$
- other constraint (e.g. from energy equations)

$$\dot{m}_g \int_{T_{og}(x_{\max})}^{T_{og}(x_{\min})} c_{p,g} dT_{og} = \dot{m}_{cl} \int_{T_{cl}(x_{\max})}^{T_{cl}(x_{\min})} c_{p,cl} dT_l$$

- numerical solution; finite differencing





Heat transfer

Other limitations of these treatments

- although these expressions for the correlations and heat coefficients are commonly used, they are not precise: large error bars expected

  - mixing processes in fuel film neglected

  - chemical reactions in boundary layer neglected

  - predictions needed: Bartz, Ambrok - predict T_w and iterate from there

  - Ambrok's equation inaccurate for strong flow acceleration

- more complex solution strategies

  - Two-Dimensional Kinetics (TDK) code: NASA, DoD

    boundary layer equations solved for real wall shape

  - Solid Rocket Performance Prediction (SPP) code